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Now the direction of motion of that point of the rod which at the instant coincides with T is clearly along the rod. Hence the instantaneous center C is on the normal to the rod at T. Again, the direction of motion of the point G on the ground is along the ground. Hence, C is in the vertical line through G.

To find the locus of C, we choose OG and OT as coördinate axes. Then,

$$x = OG = OT \cot \theta = a \cot \theta \text{ and } y = GC = TG \csc \theta = a \csc^2 \theta.$$

Eliminating θ between these two equations, we readily obtain

$$x^2 = a(y - a),$$

which is the equation of a parabola with its vertex at T $(0, a)$, its axis vertical, and latus rectum equal to a. The pole being of limited length l, the locus of C is that part of the parabola for which $0 \leqslant x \leqslant \sqrt{l^2 - a^2}$.

Also solved by H. R. Howard, William Hoover, J. B. Reynolds, and the Proposer.

342 (Mechanics). Proposed by WILLIAM HOOVER, Columbus, Ohio.

A uniform rod of length $2a$ is freely hinged at one end, at the other end a string of length b is attached which is fastened at its further end to a point on the surface of a homogeneous sphere of radius c. If the masses of the rod and sphere are equal, find the motion of the system when slightly disturbed from the vertical, and the cubic equation giving the corresponding small oscillations.

Solution by the Proposer.

For symmetry of notation, let there be three bodies of masses m_1, m_2, m_3; of axes of symmetry whose lengths are $2a_1$, $2a_2$, $2a_3$ with centers of gravity G_1, G_2, G_3 distant a_1, a_2, a_3 from corresponding extremities; radii of gyration about G_1, G_2, G_3 equal to k_1, k_2, k_3; the origin of rectangular coördinates at the fixed extremity of the highest body, (x_1, y_1), (x_2, y_2), (x_3, y_3), the horizontal through the fixed point being the axis of x; and let φ_1, φ_2, φ_3 be the angles which the axes of symmetry of the bodies make with the vertical at any time t from the beginning of motion. Then,

$$x_1 = a_1 \sin \varphi_1, \qquad y_1 = a_1 \cos \varphi_1, \tag{1}$$

$$x_2 = 2a_1 \sin \varphi_1 + a_2 \sin \varphi_2, \qquad y_2 = 2a_1 \cos \varphi_1 + a_2 \cos \varphi_2, \tag{2}$$

$$x_3 = 2a_1 \sin \varphi_1 + 2a_2 \sin \varphi_2 + a_3 \sin \varphi_3, \qquad y_3 = 2a_1 \cos \varphi_1 + 2a_2 \cos \varphi_2 + a_3 \cos \varphi_3. \tag{3}$$

The kinetic potential equation is

$$\begin{aligned} T &= \tfrac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2 + k_1^2\dot{\varphi}_1^2) + \tfrac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2 + k_2^2\dot{\varphi}_2^2) + \tfrac{1}{2}m_3(\dot{x}_3^2 + \dot{y}_3^2 + k_3^2\dot{\varphi}_3^2) \\ &= m_1 g a_1 \cos \varphi_1 + m_2 g(2a_1 \cos \varphi_1 + a_2 \cos \varphi_2) \\ &\qquad + m_3 g(2a_1 \cos \varphi_1 + 2a_2 \cos \varphi_2 + a_3 \cos \varphi_3) = V. \end{aligned} \tag{4}$$

From (1), (2), (3),

$$\dot{x}_1 = a_1 \cos \varphi_1 \cdot \dot{\varphi}_1, \qquad \dot{y}_1 = -a \sin \varphi_1 \cdot \dot{\varphi}_1, \tag{5}$$

$$\dot{x}_2 = 2a_1 \cos \varphi_1 \cdot \dot{\varphi}_1 + a_2 \cos \varphi_2 \cdot \dot{\varphi}_2, \qquad \dot{y}_2 = -2a_1 \sin \varphi_1 \cdot \dot{\varphi}_1 - a_2 \sin \varphi_2 \cdot \dot{\varphi}_2, \tag{6}$$

$$\begin{aligned} &\dot{x}_3 = 2a_1 \cos \varphi_1 \cdot \dot{\varphi}_1 + 2a_2 \cos \varphi_2 \cdot \dot{\varphi}_2 + a_3 \cos \varphi_3 \cdot \dot{\varphi}_3, \\ &\qquad \dot{y}_3 = -2a_1 \sin \varphi_1 \cdot \dot{\varphi}_1 - 2a_2 \sin \varphi_2 \cdot \dot{\varphi}_2 - a_3 \sin \varphi_3 \cdot \dot{\varphi}_3. \end{aligned} \tag{7}$$

These in (4) give

$$\begin{aligned} T &= \tfrac{1}{2}m_1(a_1^2 + k_1^2)\dot{\varphi}_1^2 + \tfrac{1}{2}m_2\{4a_1^2\dot{\varphi}_1^2 + 4a_1a_2 \cos(\varphi_1 - \varphi_2)\dot{\varphi}_1\dot{\varphi}_2 + (a_2^2 + k_2^2)\dot{\varphi}_2^2\} \\ &\quad + \tfrac{1}{2}m_3\{4a_1^2\dot{\varphi}_1^2 + 4a_2^2\dot{\varphi}_2^2 + (a_3^2 + k_3^2)\dot{\varphi}_3^2 + 8a_1a_2 \cos(\varphi_1 - \varphi_2)\dot{\varphi}_1\dot{\varphi}_2 \\ &\quad + 4a_2a_3 \cos(\varphi_2 - \varphi_3)\dot{\varphi}_2\dot{\varphi}_3 + 4a_1a_3 \cos(\varphi_1 - \varphi_3)\dot{\varphi}_1\dot{\varphi}_3\} \\ &= m_1 g a_1 \cos \varphi_1 + m_2 g\,(2a_1 \cos \varphi_1 + a_2 \cos \varphi_2) \\ &\qquad + m_3 g(2a_1 \cos \varphi_1 + 2a_2 \cos \varphi_2 + a_3 \cos \varphi_3) = V. \end{aligned} \tag{8}$$

Applying to (8) Lagrange's equations of typical form

$$\frac{d}{dt}\frac{dT}{d\dot{\varphi}_n} - \frac{dT}{d\varphi_n} = \frac{dV}{d\varphi_n}, \qquad n = 1, 2, 3, \tag{9}$$

the φ_1 function is

$$\begin{aligned}&\{m_1(a_1^2 + k_1^2) + 4a_1^2m_2 + 4a_1^2m_3\}\ddot{\varphi}_1 + 2a_1a_2m_2 \cos (\varphi_1 - \varphi_2)\ddot{\varphi}_2 + 2a_1a_2m_2 \sin (\varphi_1 - \varphi_2)\dot{\varphi}_2^2\\ &\quad + 4a_1a_2m_3 \cos (\varphi_1 - \varphi_2)\ddot{\varphi}_2 + 4a_1a_2m_3 \sin (\varphi_1 - \varphi_2)\dot{\varphi}_2^2 + 2a_1a_3m_3 \cos (\varphi_1 - \varphi_3)\ddot{\varphi}_3\\ &\quad + 2a_1a_3m_3 \sin (\varphi_1 - \varphi_3)\dot{\varphi}_3^2 = - ga_1(m_1 + 2m_2 + 2m_3) \sin \varphi_1,\end{aligned} \tag{10}$$

and the φ_2, φ_3 functions are

$$\begin{aligned}&\{m_2(a_2^2 + k_2^2) + 4a_2^2m_3\}\ddot{\varphi}_2 + \{2a_1a_2m_2 \cos (\varphi_1 - \varphi_2) + 4a_1a_2m_3 \cos (\varphi_1 - \varphi_2)\}\ddot{\varphi}_1\\ &\quad - 2a_1a_2m_2 \sin (\varphi_1 - \varphi_2)\cdot\dot{\varphi}_1^2 - 4a_1a_2m_3 \sin (\varphi_1 - \varphi_2)\cdot\dot{\varphi}_1^2 + 2a_2a_3m_3 \cos (\varphi_2 - \varphi_3)\ddot{\varphi}_3\\ &\quad + 2a_2a_3m_3 \sin (\varphi_2 - \varphi_3)\dot{\varphi}_3^2 = - ga_2(m_2 + 2m_3) \sin \varphi_2,\end{aligned} \tag{11}$$

$$\begin{aligned}&m_3\{(a_3^2 + k_3^2)\ddot{\varphi}_3 + 2a_2a_3 \cos (\varphi_2 - \varphi_3)\ddot{\varphi}_2 - 2a_2a_3 \sin (\varphi_2 - \varphi_3)\dot{\varphi}_2^2 + 2a_1a_3 \cos (\varphi_1 - \varphi_3)\ddot{\varphi}_1\\ &\quad + 2a_1a_3 \sin (\varphi_3 - \varphi_1)\cdot\dot{\varphi}_1^2\} = - m_3ga_3 \sin \varphi_3.\end{aligned} \tag{12}$$

Let φ_1, φ_2, φ_3 be so small that one may employ the approximations $\sin \varphi_1 = \varphi_1$, etc.; $\cos \varphi_1 = 1$, etc.; $\dot{\varphi}_1^2 = 0$, etc.; $\dot{\varphi}_1\dot{\varphi}_2 = 0$, etc. Then (10), (11), (12) may be written

$$\begin{aligned}&\{m_1(a_1^2 + k_1^2) + 4a_1^2(m_2 + m_3)\}\ddot{\varphi}_1 + 2a_1a_2m_2\ddot{\varphi}_2 + 4a_1a_2m_3\ddot{\varphi}_2 + 2a_1a_3m_3 \cos (\varphi_1 - \varphi_3)\ddot{\varphi}_3\\ &\qquad = - ga_1(m_1 + 2m_2 + 2m_3)\varphi_1,\end{aligned} \tag{13}$$

$$\{2a_1a_2m_2 + 4a_1a_2m_3\}\ddot{\varphi}_1 + \{m_2(a_2^2 + k_2^2) + 4a_2^2m_3\}\ddot{\varphi}_2 + 2a_2a_3m_3\ddot{\varphi}_3 = - ga_2(m_1 + 2m_3)\varphi_2, \tag{14}$$

$$m_3\{(a_3^2 + k_3^2)\ddot{\varphi}_3 + 2a_2a_3\ddot{\varphi}_2 + 2a_1a_3\ddot{\varphi}_1\} = - ga_3m_3\varphi_3. \tag{15}$$

These are the three equations of small motion in the general form of the problem.

In the present problem, $m_2 = 0$; $k_1^2 = a_1^2/3$, $k_2 = 0$, $k_3^2 = \frac{2}{5}a_3^2$; and we may put $m_1 = m_3 = m$; also $2a_1 = 2a$, $2a_2 = b$, $2a_3 = 2c$, and (13), (14), (15) are

$$\begin{aligned}\tfrac{16}{3}a\ddot{\varphi}_1 + 3g\varphi_1 + 2b\ddot{\varphi}_2 + 2c\ddot{\varphi}_3 &= 0,\\ 2a\ddot{\varphi}_1 + b\ddot{\varphi}_2 + g\varphi_2 + c\ddot{\varphi}_3 &= 0,\\ 2a\ddot{\varphi}_1 + b\ddot{\varphi}_2 + \tfrac{7}{5}c\ddot{\varphi}_3 + g\varphi_3 &= 0;\end{aligned}$$

or, using another notation of differential equations,

$$\begin{aligned}(\tfrac{16}{3}aD^2 + 3g)\varphi_1 + 2bD^2\varphi_2 + 2cD^2\varphi_3 &= 0,\\ 2aD^2\varphi_1 + (bD^2 + g)\varphi_2 + cD^2\varphi_3 &= 0,\\ 2aD^2\varphi_1 + bD^2\varphi_2 + (\tfrac{7}{5}cD^2 + g)\varphi_3 &= 0.\end{aligned}$$

Eliminate φ_1 and φ_2; then,

$$\{8abcD^6 + g(20ab + 18bc + 52ac)D^4 + g^2(80a + 45b + 63c)D^2 - 45g^3\}\varphi_3 = 0. \tag{16}$$

Let

$$\varphi_3 = L_p \cos (\lambda t + \alpha_p),$$

and after substitution in (22) and simplifying, put $\lambda^2 = \mu$, and we have

$$8abc\mu^3 - g(20ab + 18bc + 52ac)\mu^2 + g^2(80a + 45b + 63c)\mu - 45g^3 = 0,$$

the cubic equation required.

Also solved by J. B. REYNOLDS and W. E. CEDERBERG.

259 (Number Theory). Proposed by E. E. WHITFORD, College of the City of New York.

If p is relatively prime to 10, and if any multiple of p consisting of n digits has its digits permuted cyclically, the number thus formed is also a multiple of p; the number n to be determined by the congruence $10^n \equiv 1 \pmod{p}$. For example, 481, 814, and 148 are each multiples of 37.